Pooling in a predictive model of V1

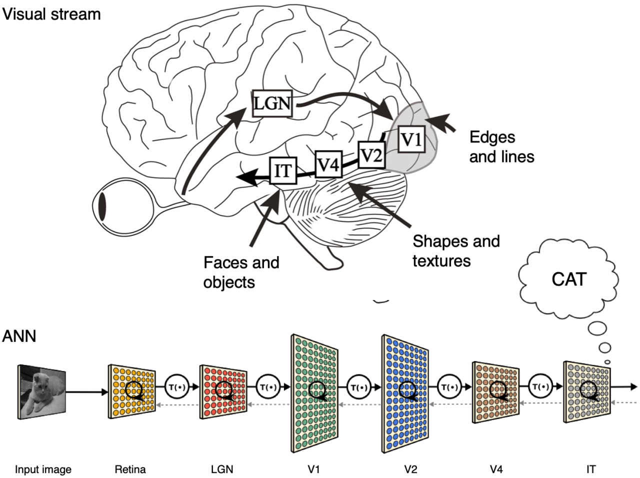
Toward understanding functional and structural diversity

Angelo Franciosini and Laurent Perrinet

Institut de Neurosciences de la Timone
CNRS and Aix-Marseille Université, France
https://laurentperrinet.github.io/

Society for Mathematical Biology Annual meeting
Minisymposium *Recent advances in mathematical neuroscience*
Cortically inspired models for vision and synaptic plasticity
June 15, 2021

Analogy between ANNs and the ventral stream in mammals



Herzog, M. H. & Clarke, A. M. Why vision is not both hierarchical and feedforward. *Frontiers in computational neuroscience* **8,** 135 (2014)

Outline

Sparse Deep Predictive Coding
- Sparse Coding
- Predictive Coding
- Sparse Deep Predictive Coding

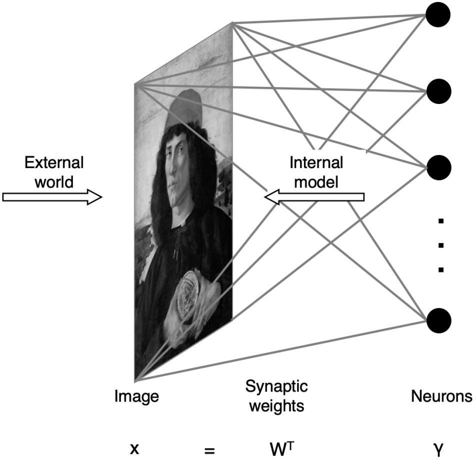
Properties of the predictive model of V1
- SDPC and the Association Field
- Pooling in the visual cortex

Conclusion: the Predictive Field
- Conclusion: the Predictive Field
- Prospect: the Predictive Field

Section 1

Sparse Deep Predictive Coding

Sparse Coding



Olshausen, B. A. & Field, D. J. Sparse coding with an overcomplete basis set: A strategy employed by V1? *Vision research* **37,** 3311–3325 (1997)

Sparse Coding

$$\boldsymbol{x} = \boldsymbol{W}^T \boldsymbol{\gamma} + \boldsymbol{\epsilon}, \quad \text{s.t. } \boldsymbol{\epsilon} \sim \mathcal{N}\left(0, \sigma^2\right) \tag{1}$$

Sparse Coding

$$\boldsymbol{x} = \boldsymbol{W}^T \boldsymbol{\gamma} + \boldsymbol{\epsilon}, \quad \text{s.t.} \ \ \boldsymbol{\epsilon} \sim \mathcal{N}\left(0, \sigma^2\right) \tag{1}$$

$$\boldsymbol{\gamma}, \boldsymbol{W} = \underset{\boldsymbol{\gamma}, \boldsymbol{W}}{\arg\min} \left(\frac{1}{2\sigma^2} \left\| \boldsymbol{x} - \boldsymbol{W}^T \boldsymbol{\gamma} \right\|_2^2 + \lambda S\left(\boldsymbol{\gamma}\right) \right) \tag{2}$$

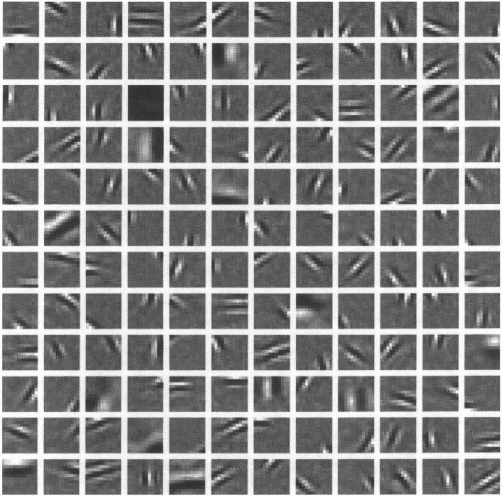
Sparse Coding

$$\boldsymbol{x} = \boldsymbol{W}^T \boldsymbol{\gamma} + \boldsymbol{\epsilon}, \quad \text{s.t.} \;\; \boldsymbol{\epsilon} \sim \mathcal{N}\left(0, \sigma^2\right) \tag{1}$$

$$\boldsymbol{\gamma}, \boldsymbol{W} = \underset{\boldsymbol{\gamma}, \boldsymbol{W}}{\arg\min} \left(\frac{1}{2\sigma^2} \left\| \boldsymbol{x} - \boldsymbol{W}^T \boldsymbol{\gamma} \right\|_2^2 + \lambda S\left(\boldsymbol{\gamma}\right) \right) \tag{2}$$

$$\boldsymbol{\gamma}, \boldsymbol{W} = \underset{\boldsymbol{\gamma}, \boldsymbol{W}}{\arg\min} \left(\frac{1}{2\sigma^2} \left\| \boldsymbol{x} - \boldsymbol{W}^T \boldsymbol{\gamma} \right\|_2^2 + \lambda \left\| \boldsymbol{\gamma} \right\|_1 \right) \tag{3}$$

Sparse Coding



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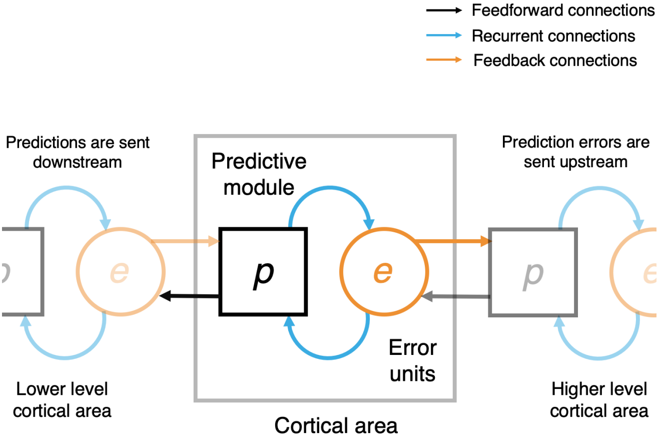
Predictive Coding



Rao, R. P. & Ballard, D. H. Predictive coding in the visual cortex: a functional interpretation of some extra-classical receptive-field effects. *Nature neuroscience* **2,** 79 (1999)

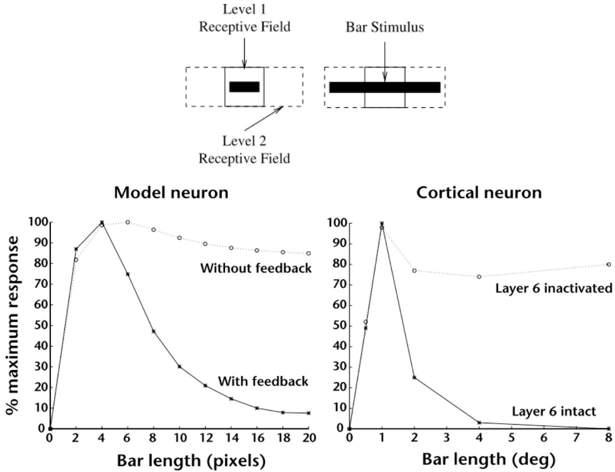
Predictive Coding

$$\begin{cases} \boldsymbol{x} = f\left(\boldsymbol{W}_1^T \gamma_1\right) + \boldsymbol{\epsilon}_1, & \text{s.t. } \boldsymbol{\epsilon}_1 \sim \mathcal{N}\left(0, \sigma_1^2\right) \\ \gamma_1 = f\left(\boldsymbol{W}_2^T \gamma_2\right) + \boldsymbol{\epsilon}_2, & \text{s.t. } \boldsymbol{\epsilon}_2 \sim \mathcal{N}\left(0, \sigma_2^2\right) \\ \ldots \\ \gamma_{N-1} = f\left(\boldsymbol{W}_N^T \gamma_N\right) + \boldsymbol{\epsilon}_N, & \text{s.t. } \boldsymbol{\epsilon}_N \sim \mathcal{N}\left(0, \sigma_N^2\right) \end{cases} \tag{4}$$

$$L = \sum_{i=1}^{N} \frac{1}{2\sigma_i} \left\| \gamma_{i-1} - f\left(\boldsymbol{W}_i^T \gamma_i\right) \right\|_2^2 \tag{5}$$

$$E = L + G + R = \sum_{i=1}^{N} \left(l_i + g_i\left(\gamma_i\right) + r_i\left(\boldsymbol{W}_i\right) \right) \tag{6}$$

Predictive Coding



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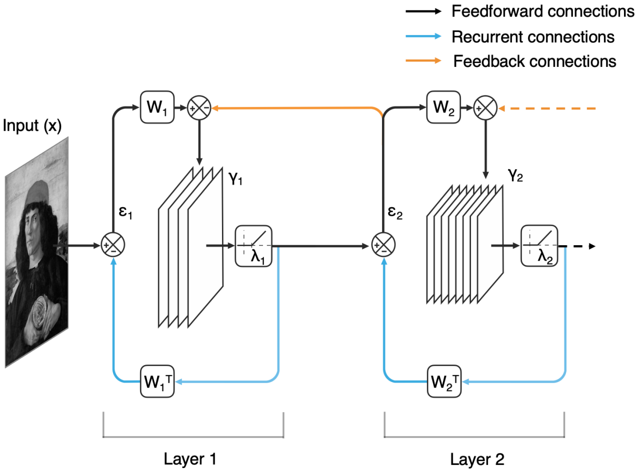
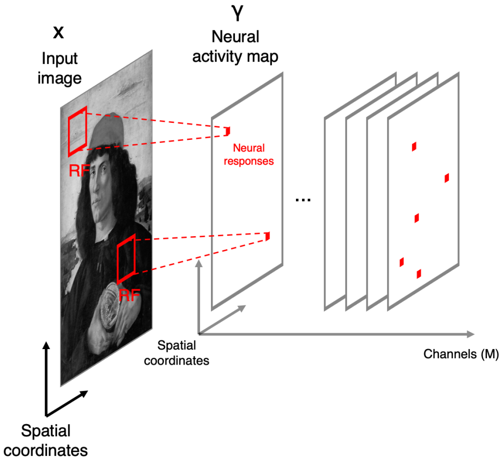
Sparse Deep Predictive Coding



Sparse Deep Predictive Coding



Sparse Deep Predictive Coding

$$\begin{cases} \boldsymbol{x} = f\left(\boldsymbol{W}_1^T \gamma_1\right) + \boldsymbol{\epsilon}_1, & \text{s.t. } \boldsymbol{\epsilon}_1 \sim \mathcal{N}\left(0, \sigma_1^2\right) \\ \gamma_1 = f\left(\boldsymbol{W}_2^T \gamma_2\right) + \boldsymbol{\epsilon}_2, & \text{s.t. } \boldsymbol{\epsilon}_2 \sim \mathcal{N}\left(0, \sigma_2^2\right) \\ \ldots \\ \gamma_{N-1} = f\left(\boldsymbol{W}_N^T \gamma_N\right) + \boldsymbol{\epsilon}_N, & \text{s.t. } \boldsymbol{\epsilon}_N \sim \mathcal{N}\left(0, \sigma_N^2\right) \end{cases} \tag{7}$$

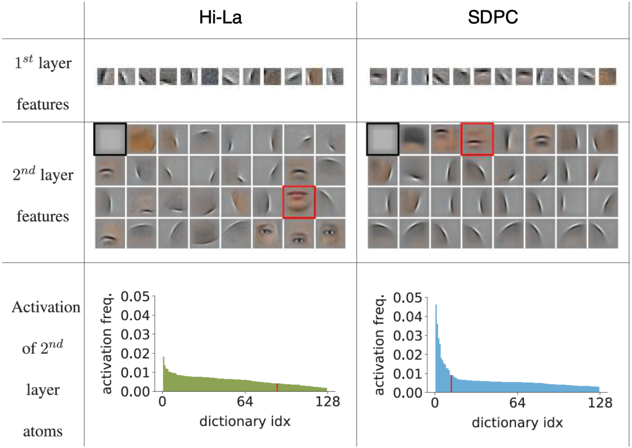
Sparse Deep Predictive Coding

$$
\begin{cases}
\underset{\gamma_1, \boldsymbol{W}_1}{\arg\min} \left(\frac{1}{2\sigma_1^2} \left\| \boldsymbol{x} - \boldsymbol{W}_1^T \gamma_1 \right\|_2^2 + \frac{1}{2\sigma_2^2} \left\| \gamma_1 - \boldsymbol{W}_2^T \gamma_2 \right\|_2^2 + \lambda_1 \left\| \gamma_1 \right\|_1 \right) \\
\dots \\
\underset{\gamma_i, \boldsymbol{W}_i}{\arg\min} \left(\frac{1}{2\sigma_i^2} \left\| \gamma_{i-1} - \boldsymbol{W}_i^T \gamma_i \right\|_2^2 + \frac{1}{2\sigma_{i+1}^2} \left\| \gamma_i - \boldsymbol{W}_{i+1}^T \gamma_{i+1} \right\|_2^2 + \lambda_i \left\| \gamma_i \right\|_1 \right) \\
\dots \\
\underset{\gamma_N, \boldsymbol{W}_N}{\arg\min} \left(\frac{1}{2\sigma_N^2} \left\| \gamma_{N-1} - \boldsymbol{W}_N^T \gamma_N \right\|_2^2 + \lambda_N \left\| \gamma_N \right\|_1 \right)
\end{cases}
\tag{8}
$$

Section 2

Properties of the predictive model of V1

SDPC and the Association Field



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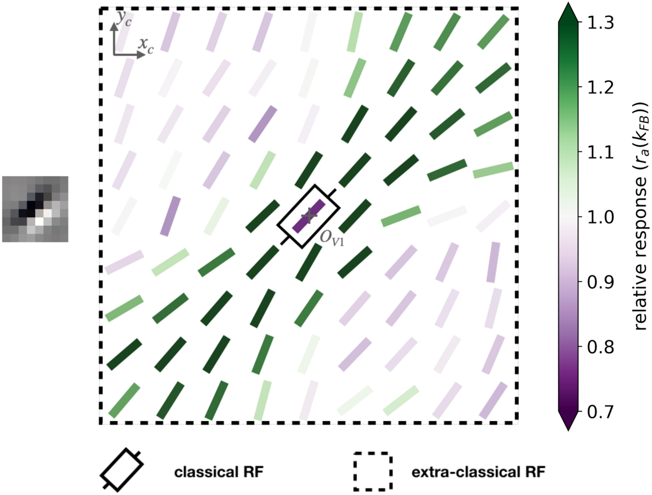
SDPC and the Association Field



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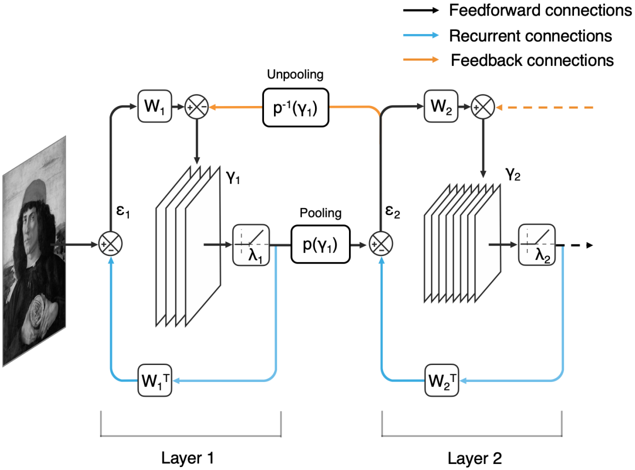
Pooling in the visual cortex

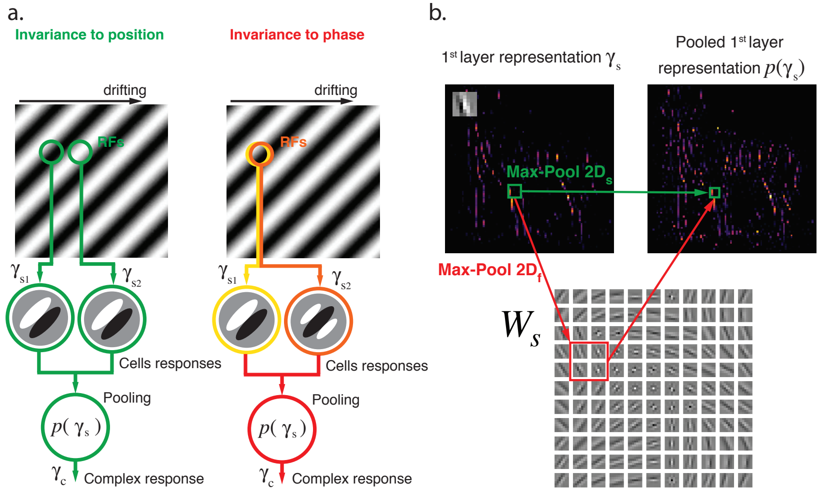


Franciosini, A. *et al.* Pooling in a predictive model of V1 explains functional and structural diversity across species. *bioRxiv.* doi:10.1101/2021.04.19.440444 (2021)

Pooling in the visual cortex

$$\begin{cases} \underset{\boldsymbol{\gamma_1}, \boldsymbol{W_1}}{\arg\min} \left(\frac{1}{2\sigma_1^2} \left\| \boldsymbol{x} - \boldsymbol{W}_1^T \gamma_1 \right\|_2^2 + \frac{1}{2\sigma_2^2} \left\| p_1(\gamma_1) - \boldsymbol{W}_2^T \gamma_2 \right\|_2^2 + \lambda_1 \left\| \gamma_1 \right\|_1 \right) \\ \ldots \\ \underset{\gamma_i, \boldsymbol{W}_i}{\arg\min} \left(\frac{1}{2\sigma_i^2} \left\| \gamma_{i-1} - \boldsymbol{W}_i^T \gamma_i \right\|_2^2 + \frac{1}{2\sigma_{i+1}^2} \left\| p_i(\gamma_i) - \boldsymbol{W}_{i+1}^T \gamma_{i+1} \right\|_2^2 + \lambda_i \left\| \gamma_i \right\|_1 \right) \\ \ldots \\ \underset{\gamma_N, \boldsymbol{W}_N}{\arg\min} \left(\frac{1}{2\sigma_N^2} \left\| \gamma_{N-1} - \boldsymbol{W}_N^T \gamma_N \right\|_2^2 + \lambda_N \left\| \gamma_N \right\|_1 \right) \end{cases} \tag{9}$$

Pooling in the visual cortex



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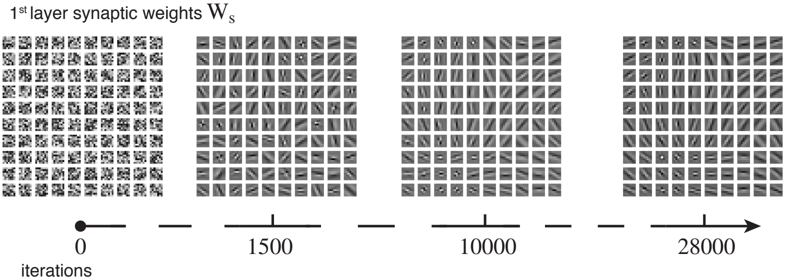
Pooling in the visual cortex



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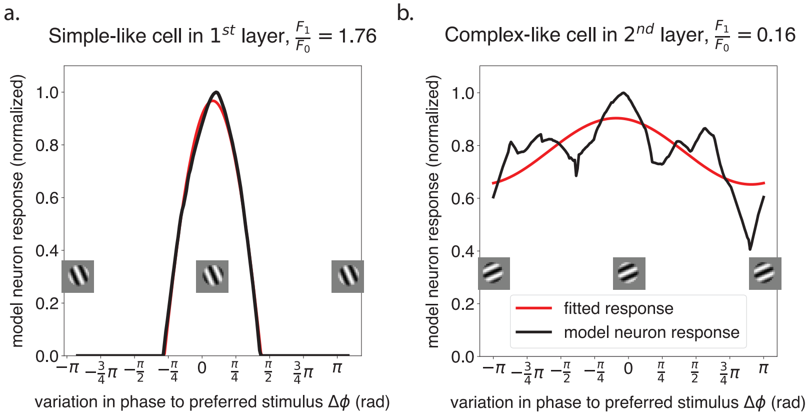

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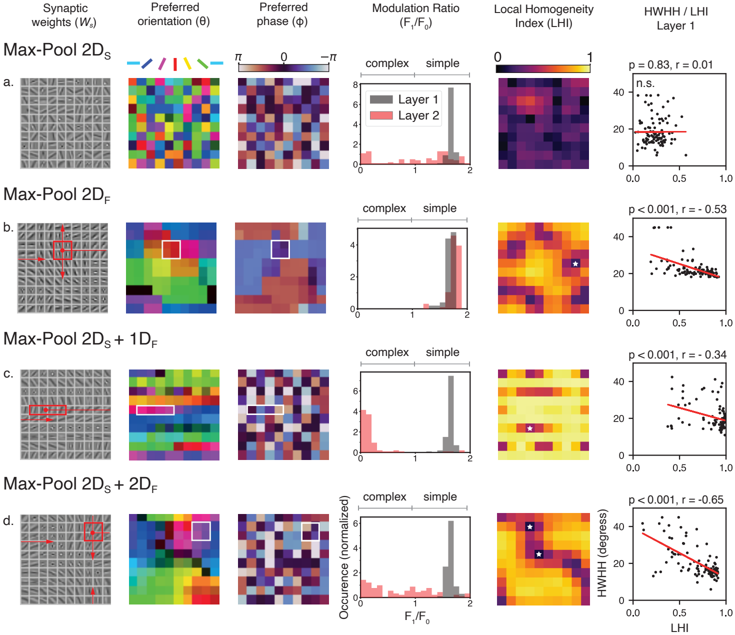


Synaptic weights (W_s)
Preferred orientation (θ)
Preferred phase (ϕ)
Modulation Ratio (F_1/F_0)
Local Homogeneity Index (LHI)
HWHH / LHI Layer 1
Max-Pool $2D_S$
a.
π
0
−π
complex
simple
Layer 1
Layer 2
p = 0.83, r = 0.01
n.s.
Max-Pool $2D_F$
b.
p < 0.001, r = - 0.53
Max-Pool $2D_S + 1D_F$
c.
p < 0.001, r = - 0.34
Max-Pool $2D_S + 2D_F$
d.
p < 0.001, r = -0.65
Occurence (normalized)
F_1/F_0
HWHH (degress)
LHI

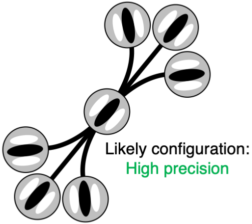
Section 3

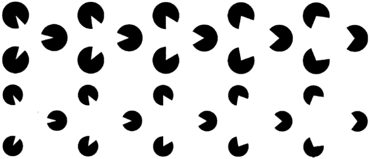
Conclusion: the Predictive Field

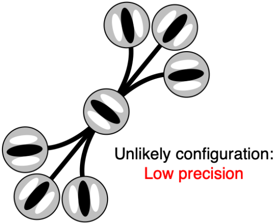
Conclusion: the Predictive Field

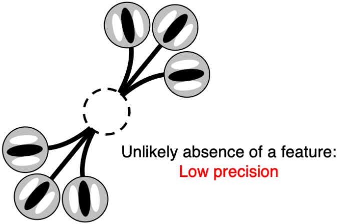
- SDPC explains different properties of the primary visual cortex;
- It explains functional and structural diversity across species;
- Lateral & feedback interactions play a crucial role in neural computations.

Prospect: the Predictive Field

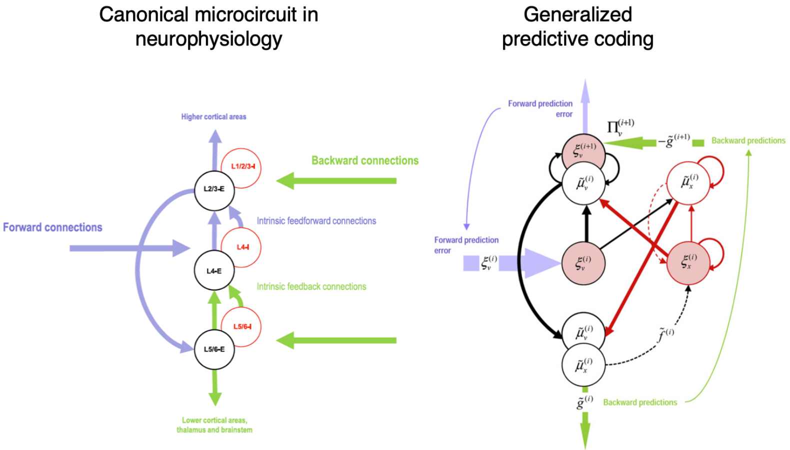








Canonical microcircuit for generalized PC



Bastos, A. M. *et al.* Canonical microcircuits for predictive coding. *Neuron* **76,** 695–711 (2012)

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1. Herzog, M. H. & Clarke, A. M. Why vision is not both hierarchical and feedforward. *Frontiers in computational neuroscience* **8,** 135 (2014).
2. Olshausen, B. A. & Field, D. J. Sparse coding with an overcomplete basis set: A strategy employed by V1? *Vision research* **37,** 3311–3325 (1997).
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References II

5. Boutin, V., Franciosini, A., Chavane, F., Ruffier, F. & Perrinet, L. Sparse deep predictive coding captures contour integration capabilities of the early visual system. *PLoS computational biology* **17**, e1008629. doi:10.1371/journal.pcbi.1008629 (2021).
6. Franciosini, A., Boutin, V., Chavane, F. & Perrinet, L. U. Pooling in a predictive model of V1 explains functional and structural diversity across species. *bioRxiv.* doi:10.1101/2021.04.19.440444 (2021).
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