Coherence detection in a spiking neuron via Hebbian learning

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Abstract

It is generally assumed that neurons in the central nervous system communicate through temporal firing patterns. As a first step, we will study the learning of a layer of realistic neurons in the particular case where the relevant messages are formed by temporally correlated patterns, or *synfire patterns*. The model is a layer of Integrate-and-Fire (IF) neurons with synaptic current dynamics that adapts by minimizing a cost according to a gradient descent scheme. This leads to a rule similar to Spike-Time Dependent Hebbian Plasticity (STDHP). Our results show that the rule that we derive is biologically plausible and leads to the detection of the coherence in the input in an unsupervised way. An application to shape recognition is shown as an illustration.

Key words: Spiking Neural Networks, Hebb Rule, Spike Time Dependent Hebbian Plasticity, Kinetic model, Rank Order Coding

1 Description of the model

1.1 Coding scheme

We will represent (as in [Gerstner99]) the signal S_i at synapse i by the sum of Dirac pulses located at the spiking times t_i^k drawn from the lists of spikes Γ_i (see Figure 1-left).

$$S_i = \sum_{k \in \Gamma_i} \delta(t - t_i^k) \tag{1}$$

Synfire patterns are generated in analogy with the response of a retina to flashed binary images. The input of the synapses is characterized as the output of single-synapse IF neurons responding to a specific binary input.This response may be described as the sum of two random point processes with different time scales. At

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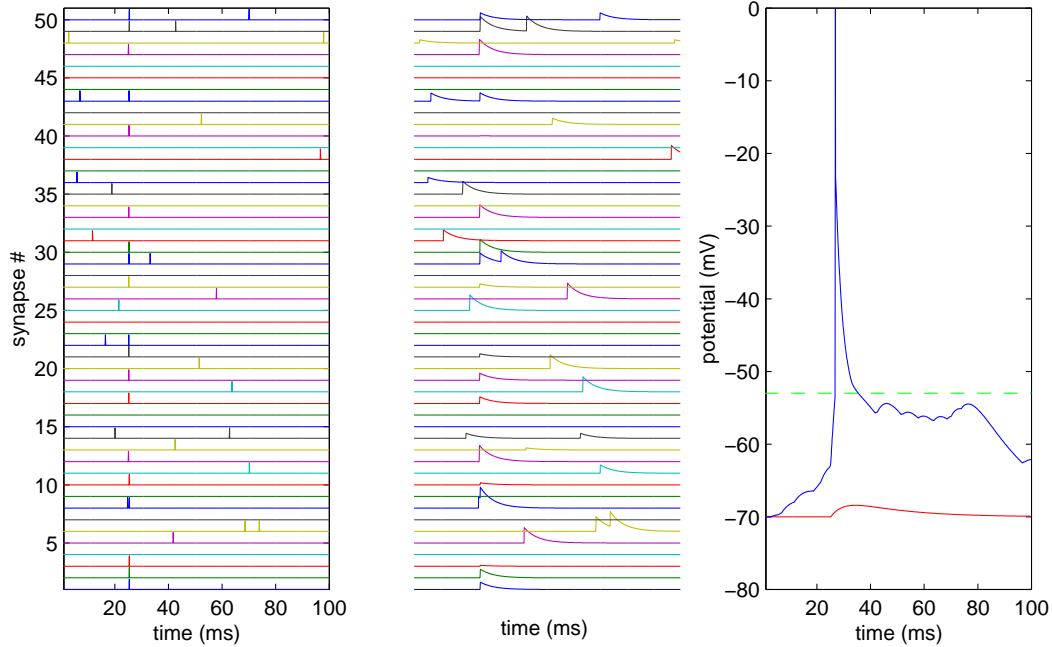*Email address:* {perrinet, samuelid}@cert.fr (Samuelides M.).

Fig. 1. Neuron Model : (left) Input spikes (with a synfire pattern at $t = 25ms$), are (middle) modulated in time and amplitude forming postsynaptic current pulses and are finally (right) integrated at the soma. When the potential (blue line) reaches the threshold (dotted green line), a spike is emitted and the potential is decreased. A sample PSP is shown in red.

a narrow time scale, the input is the spontaneous activity, i.e. a background noise independent of time and synapses that may be described by a Poisson point process of rate $1/\tau_{noise}$. At a larger time scale, the synfire pattern activates a given subset $\mathcal{M}$ of synapses once per flash with a correlation defined by its jitter τ_{jitter}. (see figure 1-left)

1.2 *Integrate-and-Fire Layer*

We will consider N_1 synapses (indexed by i) connected to a layer of N_2 neurons j. Those are generalized version of IF neurons with synaptic current dynamics, a one compartment model with no delay and the synapses have contacts characterized by their weight w_{ij}. The state variables are the $N_1.N_2$ synaptic driving conductances g_{ij} and the N_2 membrane potentials V_j. Incoming spikes trigger those conductances by opening the driving gates with time constant τ_g :

$$\frac{dg_{ij}}{dt} = -\frac{1}{\tau_g} g_{ij} + w_{ij}.S_i \tag{2}$$

and the potential V_j at the soma integrates with time constant τ_V the driving cur-

rents and the leaking current g_{leak} (with a potential $V_{rest} \approx -70\,mV$): (see Fig. 1-middle):

$$\tau_V \frac{dV_j}{dt} = g_{leak}(V_{rest} - V_j) + \left(\sum_{1 \leq i \leq N_2} g_{ij}\right) \tag{3}$$

When V_j reaches the threshold's potential $V_{threshold} \approx -54\,mV$ from below, the target neuron fires (see Fig. 1-right) and is shunted (V_j is set e.g. to $V_{reset} \approx -75\,mV$).

1.3   Reduced equations

We introduce reduced equations for this IF Layer to study its dynamical behavior and simplify its implementation. In fact, this reduction follows the concept of the Spike-Response Model (SRM) which was extensively studied in [Gerstner99,**?**]. It is similar to [Perrinet01] which aimed at reducing the STDHP equations to a set of first order equations.

$$\begin{cases} \frac{dc_i}{dt} = -\frac{1}{\tau_g}c_i + S_i \\ \tau_V \frac{dp_i}{dt} = -p_i + c_i \end{cases} \tag{4}$$

then, $V_j = V_{rest} + \left(\sum_{1 \leq i \leq N_1} w_{ij}.p_i\right)$ verifies the equation system (2, 3), with $g_{ij} = w_{ij}.c_i$.

To account for the threshold mechanism at a time t_j^k, we may then add a resetting value to V_j by setting $\eta_j(t_j^k) = V_{reset} - V_{threshold}$ and then:

$$\tau_V \frac{\eta_j(t)}{dt} = -\eta_j(t) \tag{5}$$

So that finally, an equivalent version of the IF Layer consists of (4, 5) and :

$$V_j(t) = V_{rest} + \left(\sum_{1 \leq i \leq N_1} w_{ij}.p_i(t)\right) + \eta_j(t) \tag{6}$$

This formulation depends only on the present state and not on the past values. It is therefore biologically more plausible and computationally cheaper.

Integrating these equations after emission of a presynaptic spike at t_i or a postsynaptic spike at t_j leads to :

$$c_i(t) = exp(-\frac{t - t_i}{\tau_g}) \qquad (7)$$

$$p_i(t) = \frac{\tau_g}{\tau_V - \tau_g}(exp(-\frac{t - t_i}{\tau_g}) - exp(-\frac{t - t_i}{\tau_v})) \qquad (8)$$

$$\eta(t) = (V_{reset} - V_{threshold})exp(-\frac{t - t_j}{\tau_V}) \qquad (9)$$

Those equations (6, 7, 8 and 9) are the equivalent SRM version of our IF model. More precisely eq. 7 represents the PostSynaptic Current (PSC), see fig. 1-middle, and eq. 8 the PostSynaptic Potential (PSP), see fig. 1-right. .

2 The learning mechanism

2.1 Definition of the cost function

Based on neurophysiological studies, we set the following principles :

(1) the learning is associated with a response : the nth learning step occurs at the nth output firing time t_n,

(2) to discriminate between the different input patterns, the output voltage should be close to a target value : the potential of the winning neuron (which we index $j = j_n$) should be above threshold whereas other neurons should be hyperpolarized,

(3) economy of the total synaptic efficacy and current use should be respected.

A possible cost function may therefore be the squared distance to the potentials of neurons at the firing time t_n added to the total sum of the squared weights:

$$2.E = \sum_{1 \leq j \leq N_2}(V_j - V_j^t)^2 + \alpha \sum_{1 \leq j \leq N_2}(\frac{\partial V_j}{\partial t})^2 + \beta \sum_{1 \leq i \leq N_1}^{1 \leq j \leq N_2}w_{ij}^2 \qquad (10)$$

$$V_j^t = V_{rest} \qquad for \qquad j \neq j_n \qquad (11)$$

$$V_{j_n}^t = V_{threshold} + \Delta V \qquad (12)$$

Where α and β are scaling parameters and $\Delta V \approx 5\,mV$.

2.2 Gradient descent

It follows from equations 10 and 6 :

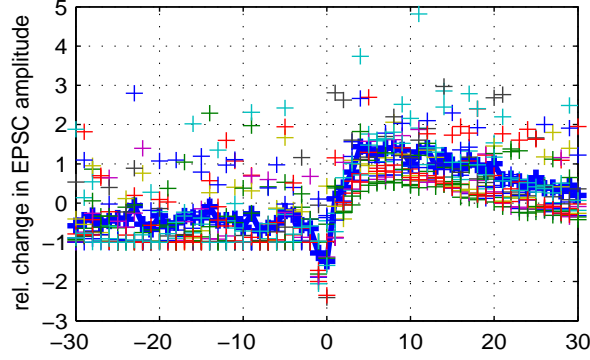Fig. 2. STDHP : Relative weight change versus time difference between input and output spikes in a single neuron modelized with our model.

$$\frac{\partial E}{\partial w_{ij}} = (V_j - V_j^t)\frac{\partial V_j}{\partial w_{ij}} + \alpha\frac{\partial V_j}{\partial t}\frac{\partial}{\partial w_{ij}}\frac{\partial V_j}{\partial t} + \beta w_{ij}$$

$$= (V_j - V_j^t).p_i + \alpha\frac{\partial V_j}{\partial t}.\dot{p}_i + \beta w_{ij}$$

We may therefore formulate the gradient descent algorithm in our model as :

$$w_{ij}^{n+1} = w_{ij}^n - \gamma_n.\frac{\partial E}{\partial w_{ij}}$$

$$= (1 - \beta_n)w_{ij}^n + \gamma_n(V_j^t - V_j).\frac{\partial V_j}{\partial w_{ij}} + \alpha_n.\frac{\partial V_j}{\partial t}.\frac{\partial \dot{V}_j}{\partial w_{ij}}$$

with learning factors $\gamma_n, \alpha_n and \beta_n$ which satisfies (e.g. for γ_n) $\sum_{n=1...\infty} \gamma_n \rightarrow \infty$ and $\sum_{n=1...\infty} \gamma_n^2 < \infty$.

Finally,

$$w_{ij}^{n+1} = (1 - \gamma_n)w_{ij}^n + \alpha_n(V_j^t - V_j).p_i + \beta_n(\sum w_{ij}.\dot{p}_i)\dot{p}_i \qquad (13)$$

2.3 Spike-time Dependent Plasticity

A closer look at equation 13 shows a direction in the change of w_{ij} is proportional to $(V_j^{target} - V_j).p_i$. This is a hebbian type of rule : when a neuron j fires after the firing of synapse j, there is a mechanism that strengthen the connection. The strengthening depends therefore on the relative time of the pre- and post-synaptic spikes (see fig. 2) as is observed in biological systems [Bi98].
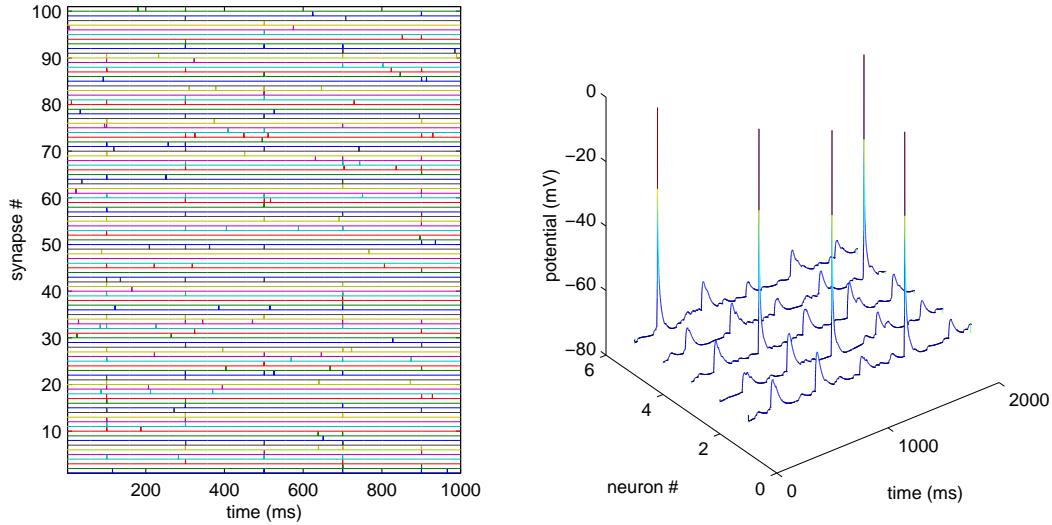
Fig. 3. Coherence detection: (left) different input patterns ($t = 100ms, 300ms, 500ms, 700ms, 900ms$) are (right) learnt by the system : only one neuron per input fires (100 learning steps)

3 Numerical results

We implemented this model using discrete versions of the differential equations (forward Euler method) on a MATLAB system.

3.1 Response to synfire patterns

To achieve this experiment we presented synfire patterns to the layer. The weights were set at random so that the network could fire to all the inputs. The patterns were presented at random times that were sufficiently distant. This unsupervised learning converges quickly, and as may be observed in neuromuscular connectivity, the synapses tend to sparsify and the neurons tend to respond to only one input (see Figure 3).

3.2 Response to oriented bars

The next experiment consisted in applying those results to a basic retina which input consists of centered rotated lines. A fixed analogical contrast layer (ON and OFF radial cells) sends then spikes to the learning layer that adapts with the rule we presented. We observe unsupervised emergence of V1-like receptors fields sensitive to the orientation (see Figure 4). Further experiments with lateral interactions and accounting for dendritic delay show even more realistic filters and column architecture.
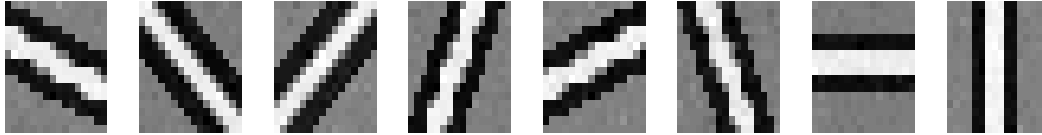
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Fig. 4. Oriented bars detection: after learning, the weights show sensitivity to orientation (black:OFF; white:ON; gray:neutral)

Conclusion

We have presented an original gradient descent method to find a learning rule for a layer of spiking neurons. The simplicity of the rule gives a new insight into the comprehension of the mechanism behind the observed STDHP. Further work is done for the detection of asynchronous patterns.

However, this study should be extended to more realistic spike trains (e.g. bursts), account for more complex behavior (e.g. facilitation and depression) and may be extended to population of neurons and recurrent systems.

Online simulations

http://laurent.perrinet.free.fr/app/app.html

References

[Bi98] Bi G.-q., Poo M.-m. (1998). *Synaptic modifications in cultured hippocampal neurons: dependence on spike timing, synaptic strength, and postsynaptic cell type*, J. Neurosci., 18:10464–10472.

[Gerstner99] Gerstner, W. (1999). *Spiking neurons.* In Pulsed Coupled Neural Networks, W. Maass, and C. M. Bishop, chapter 1, pages 3–54. MIT Press, Cambridge.

[Kempter99] Kempter, R., Gerstner, W. and van Hemmen, J. (1999). *Hebbian learning and spiking neurons.* Physical Review E, 59(4):4498-4514.

[Perrinet01] Perrinet L., Delorme A. and Thorpe S. (2001). *Network of integrate-and-fire neurons using Rank Order Coding A: How to implement spike timing dependent plasticity.* Neurocomputing, 38-40, pages 817–822.