# A generative model for Spike Time Dependent Hebbian Plasticity

Laurent Perrinet (*perrinet@cert.fr*)
ONERA/DTIM, 2, avenue É. Belin, 31055 Toulouse cedex, France

## 1 Spiking Neural Networks and the Hebb Rule

Based on neurophysiological observations on the behavior of synapses, Spike Time Dependent Hebbian Plasticity (SDTHP) is a novel extension to the modeling of the Hebb Rule. This rule has enormous importance in the learning of Spiking Neural Networks (SNN) but its mecanisms and computational properties are still to be explored. Here, we present a generative model for SDTHP based on a simplified model of the synaptic kinetic.
Due to its simplicity and biological plausibility, we have chosen to implement an hebbian-like learning based on the temporal relation between pre- and post-synaptic firing. We may reformulate it in the context of SNNs as a *temporal hebb rule* : "When there exist synapses between two neurons, the ones transmitting presynaptic spikes until the emission of the postsynaptic spike are reinforced, the others weakened."
To model it, we first used a binary rule that reinforced (resp. inhibited) equally the synapses activated before (resp. after) the postsynaptic spike. Actually, biological learning mechanisms depend on very small difference in the latencies between the upper and the downstream spiking date ([2] and [1]). In these last experiences, repeated pairing of a pre- and a post-synaptic spike was applied to a synapse leading to potentiation and depression similar to the temporal hebb rule. More precisely, the amplitude of the relative change in the weights was exponentially decreasing with the absolute spike time difference on a time scale of $\approx 20$ ms. We present here a general synaptic model for SDTHP.

## 2 A generative model for STDHP

**Description of the model** In our model, we will consider that neurons are emitting spikes that propagate along the axon but also back to the dendrites. Synapses are characterized by a weight $g.g_{max}$ where $g_{max}$ is the maximum possible weight and $g$ ranges from 0 to 1. On one side, we'll consider a pool of emitters quantified by their relative concentration $C$. This quantity is triggered by presynaptic spikes but this pool is limited. On the other side, we'll consider a pool of receivers quantified by $D$ and mediated by postsynaptic spikes.
**Modeling the synapses' dynamics by kinetic equations** Modeling $C$ (resp. $D$) for the synapse $(A-B)$ (from neuron A to neuron B) with a first order kinetic pulse based model of decay time constant $\tau_C$ and pulse amplitude $\alpha_C$ (resp. $\tau_D$ and $\alpha_D$), we get by writing the presynaptic spike times by $t_k^{(A-B)}$ (resp. postsynaptic by $t_k^{out}$) :

$$\begin{aligned} \frac{dC^{(A-B)}}{dt} &= -\frac{1}{\tau_C} C^{(A-B)} + \alpha_C \sum_k \delta(t - t_k^{(A-B)}).(1 - C^{(A-B)}) \\ \frac{dD}{dt} &= -\frac{1}{\tau_D} D + \alpha_D \sum_j \delta(t - t_j^{out}).(1 - D) \end{aligned}$$

Finally, according to our definition of synaptic weight, we may model the variation of $g$ like the combination of an excitation relative to the synapse strength and proportional to the emitters' concentration when a postsynaptic spike arrives and an inhibition relative to the existing synapse weakness and proportional to receivers' concentration when a presynaptic spike arrives. Modeling it with a first order kinetic of decay time constant $\tau_g$ we get :

$$\tau_g \frac{dg^{(A-B)}}{dt} = +(1 - g^{(A-B)}).C. \sum_j \delta(t - t_j^{out}) - g^{(A-B)}.D. \sum_k \delta(t - t_k^{(A-B)})$$

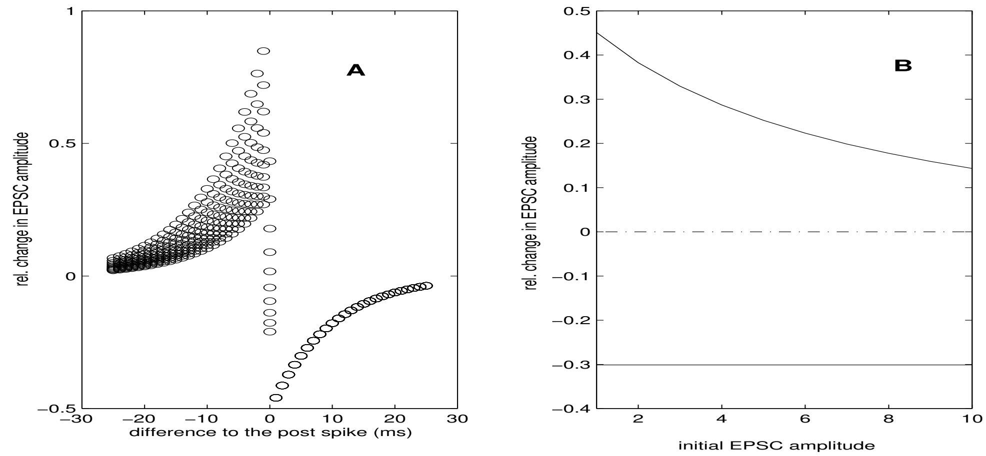


Figure 1: Simulation of the synapse model. A : relative weight variation for different initial weights. The conditions of the simulation are replicated from [2]. The weight are strengthened if the presynaptic spike occurs before the postsynaptic spike. The amplitude of change decreases exponentially with the time difference. B : relative weight variation depending on the initial weight (replicating the conditions of [1] i.e. a time difference of $\pm 6ms$). The rule is multiplicative : decreasing with weight amplitude for potentiation and constant for depotentiation.

# 3 Results

**Fitting neurophysiological results** Replicating the conditions of [2] we may easily obtain for a given initial weight an analytical formulation for the relative change of weight in our model. The constant of our model (the time constants and amplitudes of $C$ and $D$) correspond to the time and amplitude constants of the explicit rule respectively before and after the postsynaptic spike (see Fig. 1-A). We also find that the resulting explicit expression of weight change according to the initial weight (see Fig. 1-B) is analogous to the multiplicative rule described in [1] and that is analytically studied in [3].
**Rule's properties** The rule we propose is therefore a natural extension of the explicit rules described in [3] and [4]. We applied it to different models of neurons like Integrate-and-Fire, obtaining results similar as those in [3] and [4]) :

- Adaptation to random input : given a random input (like a pseudo-Poisson spike train), the neuron tends to generate a spike train with a great variability in the interspike interval (ISI) distribution,
- Learning to detect a synchronized input : The rule is adapted to cluster synapses that receive synfire inputs, leading to a bimodal weight distribution differentiating synchronized and unsynchronized inputs.
- Asynchronous wave detection : under certain conditions, this rule may provide a way to detect temporal patterns.

To conclude, faced with unsatisfactory models of the mecanisms underlying Spike Time Dependent Plasticity [4], we found a generative model that is compatible with electrophysiological results and is able to explain the relation between synapse strength and percentage of change in synaptic weight [1].

# References

[1] Bi G.-q., Poo M.-m. (1998), *Synaptic modifications in cultured hippocampal neurons: dependence on spike timing, synaptic strength, and postsynaptic cell type.* J. Neurosci., 18:10464–10472.

[2] Markram H. and J. Lübke and M. Frotscher and B. Sakmann (1997) *Regulation of Synaptic Efficacy by Concidence of Postsynaptic APs and EPSPs* Nature, 275, 213-215.

[3] Rubin J., Lee D. and Sompolinsky H. (2000) *Equilibrum Properties of Temporally Asymmetric Hebbian plasticity.* ESF Symposium on Neuronal Plasticity and Dynamics, Trieste.

[4] Song S., Miller K. D., Abbott L. F. (2000) *Competitive Hebbian learning through spike time dependent synaptic plasticity.* Nat. Neurosci., 3:919–926.