A Hierarchical, Multi-Layer Convolutional Sparse Coding Algorithm based on Predictive Coding

A. Franciosini[1,*], V. Boutin[1,2], L. Perrinet[1]

[1]Aix Marseille Univ, CNRS, INT, Marseille, France, [2]Aix Marseille Univ, CNRS, ISM, Marseille, France

*Contact: angelo.franciosini@univ-amu.fr

Introduction

- Predictive Coding (PC) is an influential framework introduced by Rao & Ballard [1] to model neural processes in the primary visual cortex of mammals (V1). PC exploits the hierarchical structure of sensory information into a bi-directional update scheme: Higher-level cortical layers predict at best the activity of the lower-level ones and send the prediction through **feedback connections** [2].
- PC gives a possible explanation to **extra-classical** receptive fields effects in V1 [1], this is also in line with the abundance of feedback connectivity in the brain [3].
- When implemented in a **recurrent neural network**, with the addition of sparsity constraints, PC can explain the emergence of edge sensitive cells in low-level visual areas as well as more specific descriptors in higher cortical areas [4].
- We show that this model, called Sparse Deep Predictive Coding network (SDPC), can also account for the **topological organization** of the primary visual cortex when imposing a max-pooling operator operator across small groups of neurons. Moreover, we show that the resulting model encodes for edges of specific orientation independently of their phase, a behaviour analogous to the one observed in neural recordings of **complex cells** [5].

Method

SDPC Network for simple and complex cells modeling:

We implemented a model of cortical area V1 as a neural network composed of two layers, implementing simple and complex cells respectively. Our algorithm combines the architecture of a convolutional neural network with the predictive coding model proposed by Rao & Ballard [1] (see Fig. 1a) into a Sparse Deep Predictive Coding network (SDPC) architecture [4]. We introduce a pooling function where, in addition to a spatial pooling, the maximum activity is selected across small groups of neurons from neighboring channels, enforcing a $2D$ topology. The competition mechanism introduced by this operators enforces neighbouring neurons within this topology to encode for similar features. This functional organization induces some degree of tolerance with respect to small variations in the input image, making the complex-cells model account for non-linear relationships with the presented stimulus.

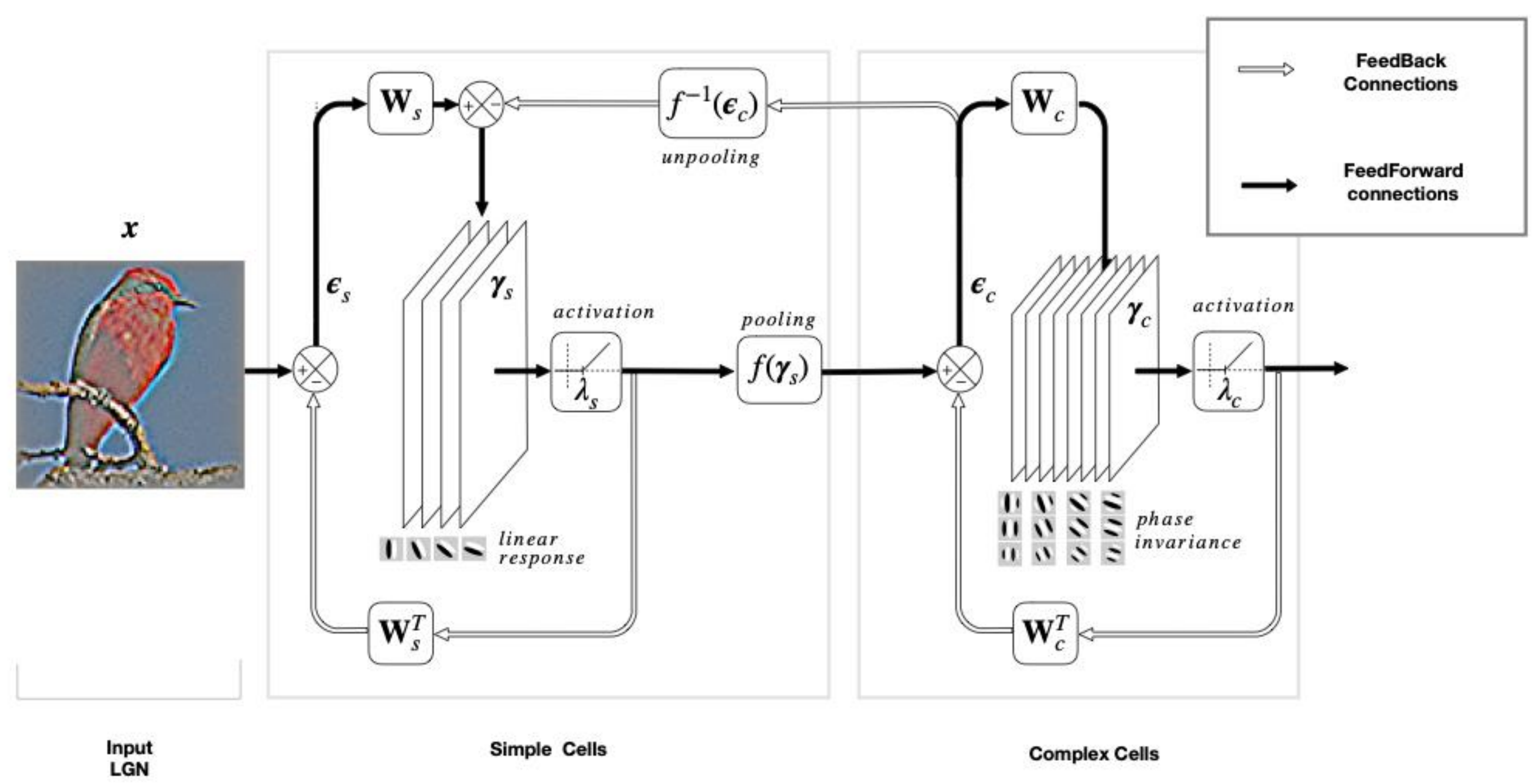

Figure 1: Simplified state update scheme of the SDPC: black and white arrows indicate respectively feedforward and feedback connections. $\boldsymbol{x}$ is the input image, $\boldsymbol{\gamma}_s$ and $\boldsymbol{\gamma}_s$ are neural activity maps, $\boldsymbol{W}_s$ and $\boldsymbol{W}_c$ the convolutional kernels. The function f is a spatial transformation between layers that regulates the shape of feedback information flow.

Mathematical formulation:

$$\mathcal{L} = \frac{1}{2}\|\boldsymbol{x} - \boldsymbol{W}_s^T \circledast \boldsymbol{\gamma}_s\|_2^2 + \frac{1}{2}\|f(\boldsymbol{\gamma}_s) - \boldsymbol{W}_c^T \circledast \boldsymbol{\gamma}_c\|_2^2 + \lambda_s\|\boldsymbol{\gamma}_s\|_1 + \lambda_c\|\boldsymbol{\gamma}_c\|_1 \quad (1)$$

Acknowledgments

This work was supported by the Ph.D. program in Integrative and Clinical Neuroscience. It received funding by the Aix-Marseille Université through the "Excellence Initiative" (A*MIDEX) , see `https://laurentperrinet.github.io/project/phd-icn`.

Results

Receptive Fields learned from Natural Images:

Fig. 2 shows the 1^{st} layer of kernels after learning. In presence of a classical spatial pooling the network is able to extract localized edge detectors (Fig. 2a) analogous to the receptive fields of simple cells in V1 [6]. In this case, the disposition of the filters is invariant to permutations of the channels. Fig. 2b, on the other hand, shows the organization that emerges when a topological structure is imposed on the pooling function. Enforcing the pooling across neighbouring channels constrains neighbouring kernels to encode for similar features. In particular, edge-like filters with similar orientation and phase tend to be grouped in neighboring channels. This organization shows qualitatively strong similarities with the formation of macro-columns structures as found in V1, for which edges of similar orientation, frequency, and color are shown to be encoded by groups of neighbouring neurons around a pinwheel [7].

(a)

(b)

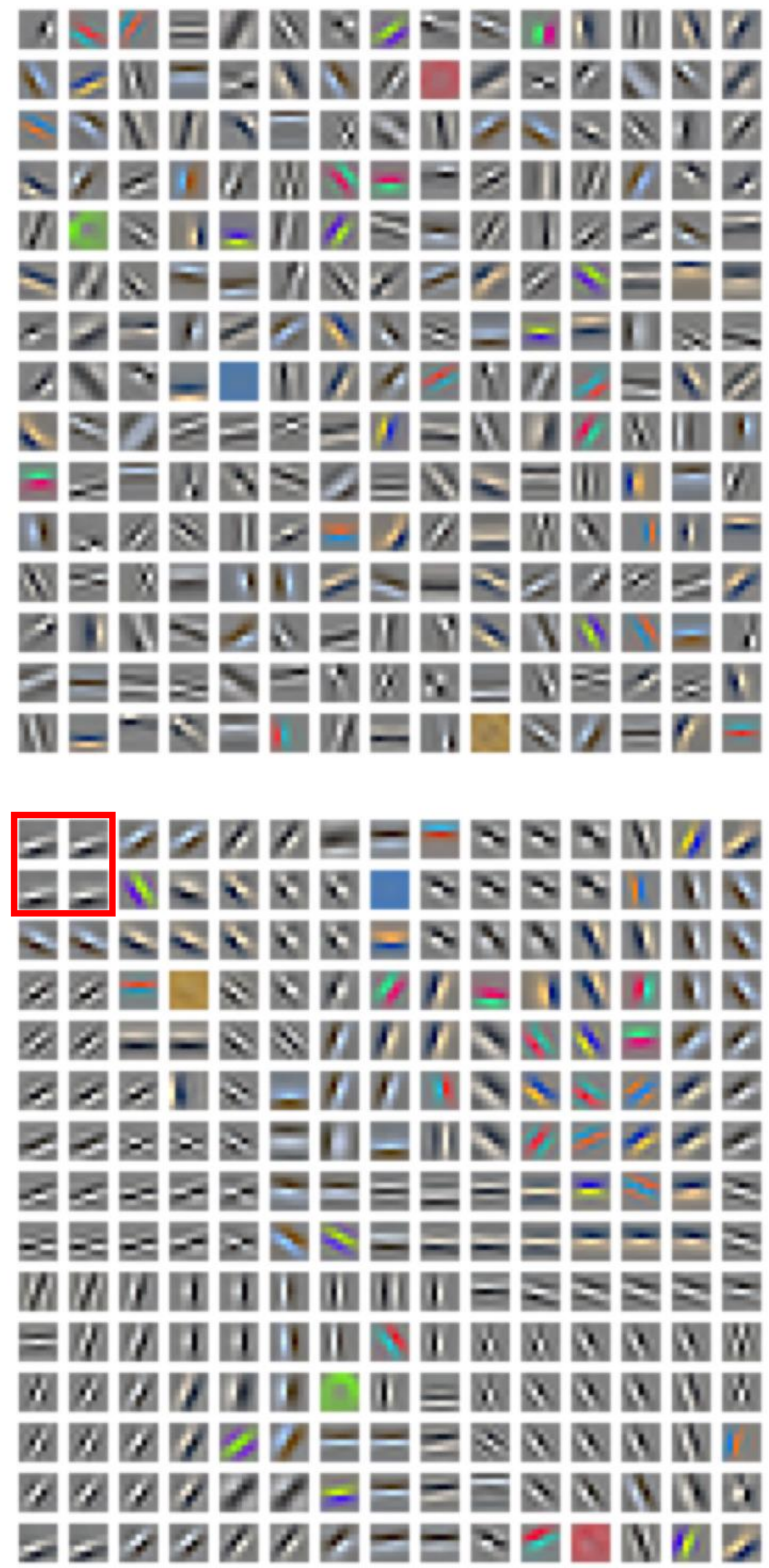

Figure 2: (a) Emergence of Gabor-like kernels $\boldsymbol{W}_s$ in the 1^{st} layer of the network (225 kernels, 7×7 pixels) learned from the STL10 database [8]. Their relative position in the grid is random. (b) Kernels learned when enforcing a topological structure. The red square indicates one of the pooling groups.

Results

Complex Cells-like invariance to phase:

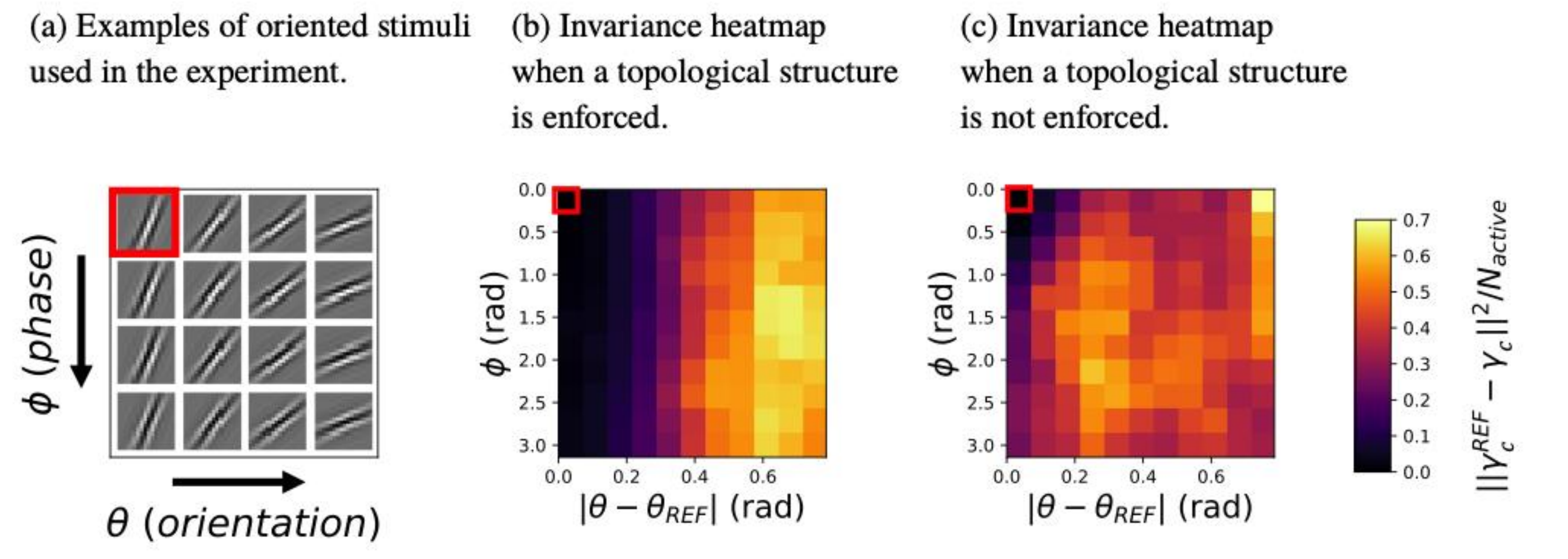

Figure 3: Variation in network activity for different phases and orientations of the input stimulus with respect to a reference edge. (a) example of different stimuli generated by varying the phase ϕ, in the range $\{0, \pi\}$, and the angle θ, in the range $\{\frac{1}{8}\pi, \frac{3}{8}\pi\}$. We show the change in activity respectively for a network showing a topological structure (b) and for a network with no topological structure (c).

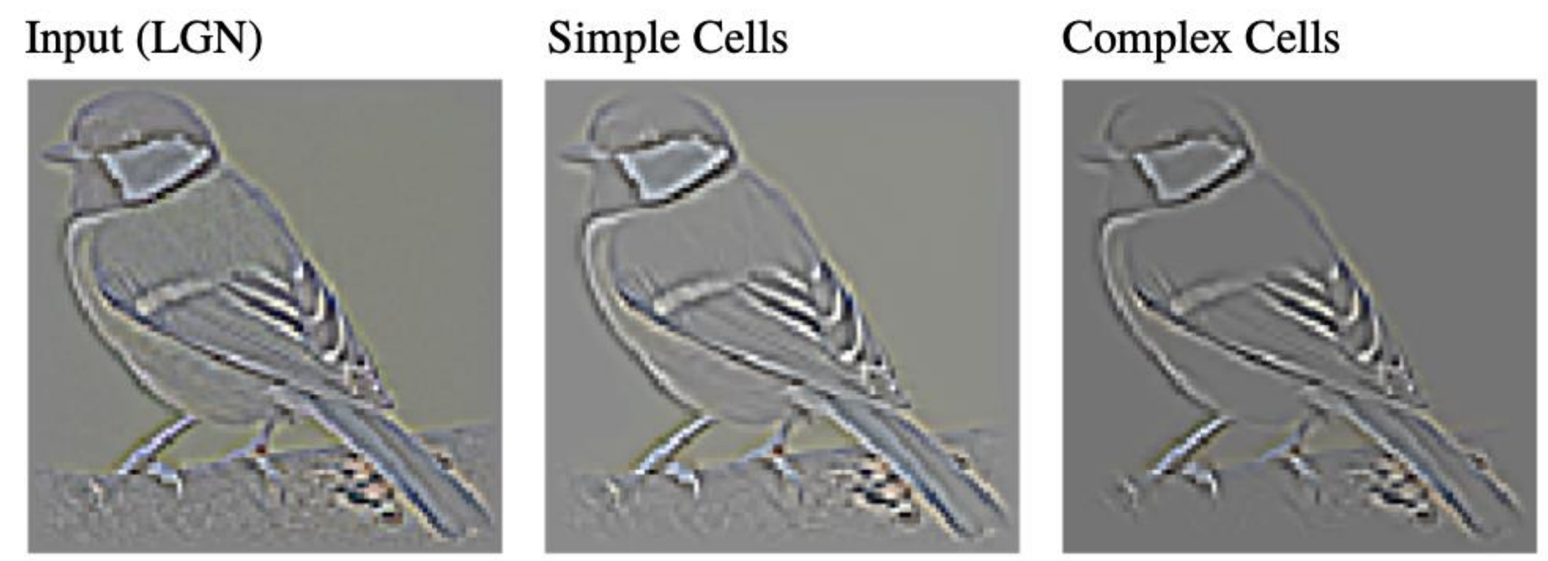

Figure 4: Example of two stimuli that maximize the activity of the simple cells (center) and the complex cells layers (right), with the reference stimulus (left).

Conclusion

- We showed that a two-layered SDPC model of V1 can predict **highly non-linear** cell behaviors observed in the mammals' visual cortex [5].
- Such a behaviour is likely of key importance for implementing **object recognition** in a neural substrate. Indeed, natural images are known to be efficiently described by co-linear and co-circular sets of edges described by Gabor filters of similar orientation disposed along smooth trajectories [9].
- Complex cells are likely implementing a **hierarchical model** [7] by exploiting such regularities together with other functional structures of the visual cortex.

References

[1] Rajesh PN Rao and Dana H Ballard. "Predictive coding in the visual cortex: a functional interpretation of some extra-classical receptive-field effects". In: *Nature neuroscience* 2.1 (1999), p. 79.

[2] Stewart Shipp. "Neural elements for predictive coding". In: *Frontiers in psychology* 7 (2016), p. 1792.

[3] Hillel Adesnik and Alexander Naka. "Cracking the Function of Layers in the Sensory Cortex". In: *Neuron* 100.5 (2018), pp. 1028–1043.

[4] Victor Boutin et al. "Meaningful representations emerge from Sparse Deep Predictive Coding". In: *arXiv preprint arXiv:1902.07651* (2019).

[5] Jan Antolik and James A Bednar. "Development of maps of simple and complex cells in the primary visual cortex". In: *Frontiers in computational neuroscience* 5 (2011), p. 17.

[6] Bruno A Olshausen and David J Field. "Sparse coding with an overcomplete basis set: A strategy employed by V1?" In: *Vision research* 37.23 (1997), pp. 3311–3325.

[7] David H Hubel. "Brain and visual perception". In: *Perception*. Vol. 34. 2005, pp. 3–3.

[8] Adam Coates, Andrew Ng, and Honglak Lee. "An analysis of single-layer networks in unsupervised feature learning". In: *Proceedings of the fourteenth international conference on artificial intelligence and statistics*. 2011, pp. 215–223.

[9] Laurent U Perrinet and James A Bednar. "Edge co-occurrences can account for rapid categorization of natural versus animal images". In: *Scientific reports* 5 (2015), p. 11400.